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A statistical test of the two reservoir model for helium isotopes

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Abstract

A common hypothesis is that helium isotopes in mantle-derived materials represent two distinct populations, the midocean ridge (MORB) reservoir and the oceanic island (OIB) reservoir. We ask the question 'Can we disprove the null hypothesis that OIB and MORB samples are drawn from the same population distribution function?' The answer is no, meaning that the datasets could be drawn from the same population. Previous studies have reached the opposite conclusion by comparing extreme values in one population with the mean value, or range, of the other population after the extreme values are removed. The variance of the $^3He/^4He$ distribution for various ridge segments decreases with the spreading rate and the high variance of individual OIB datasets is consistent with slow spreading, small sampled volume, or low degrees of melting or degassing.

Keywords: helium; statistical analysis; isotope ratios; basalts; heterogeneity

1. Introduction

The standard model of chemical geodynamics involves two reservoirs, one for midocean ridge basalts (MORB) and one for ocean island basalts (OIB). Apparently, the first one is quite homogeneous and the other one is very heterogeneous [1–3].

The MORB reservoir is thought to be homogeneous because some isotopic ratios show less scatter in ridge basalts than in OIB. The common explanation is that MORB are derived from a well-stirred convecting part of the mantle while OIB are derived from a different, deeper reservoir. Alternatively, the homogeneity of MORB could be attributed to the sampling process [4,5].

Ridges process large volumes of the mantle and involve large degrees of melting. A consequence of the Central Limit Theorem (CLT) is that the variance of samples drawn from a heterogeneous population (reservoir) depends inversely on the sampled volume [4–7]. Homogeneity of a sample population (say, all MORB samples) can simply reflect the integration effect of large volume sampling. The apparent homogeneity of MORB is partly an artifact. Samples which are thought to be contaminated by plumes are simply removed from the dataset prior to statistical analysis. Sometimes the definition of plume influence is arbitrary; for example, isotopic ratios which exceed an arbitrary cutoff are trimmed from the data.

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The MORB reservoir appears to be particularly homogeneous for helium isotopes since $^3He/^4He$ ratios (R) show little dispersion for ridge basalts, although various ridges have different values and variances that depend on spreading rate and ridge maturity [2,4,5]. A stringent test of the independent-isolated reservoir hypothesis can now be undertaken with datasets that have been compiled for these hypothetical sources ([2,4,5]; D. Graham, personal communication (2000), in [4]).

The alternative hypothesis is that the mantle is heterogeneous on various scales [4,5] and that midocean ridge and oceanic island volcanoes sample this mantle in different ways and to different extents.

We test the hypothesis H_0 that OIB samples are drawn from the same population as MORB samples. The alternative hypothesis H_1 is that OIB and MORB samples represent distinctly different populations. If H_0 is true then the samples can be regarded as samples of the same reservoir. We show that H_1 can be rejected with 95% confidence.

2. Data

There are several recent compilations of the $^3He/^4He$ ratio (R) in OIB and MORB (Table 1). MORB1 [4] includes all data along the global spreading ridge system including backarc basins, near-ridge seamounts, new ridges and subaerial exposures of the ridge. MORB2 includes only submarine samples along the midocean ridge crest; samples which are thought to be contaminated are removed from the dataset (D. Graham, personal communications (2000), in [4]). MORB3 is filtered by discarding samples from shallow depth (<2.5 km) or with isotopic ratios greater than 11 R_A [4,5]. MORB4 is an independent dataset [2] filtered to remove samples thought to be influenced by plumes. These last two datasets are representative of datasets trimmed to remove a hypothetical extraneous component. Hypothesis tests cannot ordinarily be done on such processed data.

OIB samples show great diversity and exhibit values both higher and lower than found along most ridges. A recently published compilation is used [2] for the present calculations. A much larger compilation for OIB [8] has not been analyzed statistically but the median, a robust measure of central tendency, of the two populations is the same (8.5–9.0 R_A). Note that (Table 1) the mean $^3He/^4He$ ratio of basalts from the global spreading ridge system exceeds the OIB mean. Attempts to remove plume influence have actually moved the filtered data closer to the OIB mean.

Geochemical variations in a well-sampled system such as a midocean ridge or an oceanic island can be characterized by an average value, or mean, and a measure of dispersion such as the standard deviation or variance. When dealing with isotopic ratios the appropriate measures of central tendency are the medians and the geometric means, since these are invariant to inversion of the ratio. Likewise, when dealing with ratios the appropriate averages involve the absolute concentrations in addition to the ratios, that is, the ratios must be weighted appropriately before being combined [5]. However, the unweighted arithmetic means are usually used in geochemical calculations and this practice is followed here since some of the datasets are presented only in that form [2].

3. Procedure

Statistical tests have been devised to assess the significance of the differences of the means and standard deviations of samples drawn at random from the same population, e.g. [6]. Standard tests (sometimes called z-tests) are used to test whether the differences of the means of various populations (Table 1) differ significantly from zero. These tests show that at the 95% confidence level the means of all the MORB datasets are the same as the OIB mean. That is, MORB and OIB are part of the same population.

The F-test is then applied. The hypothesis that MORB and OIB do not come from the same population is confidently (95%) rejected for MORB1, the unfiltered spreading ridge dataset. For the other (filtered) MORB datasets the hypothesis cannot be rejected. A hypothesis cannot be tested using data that have been trimmed by use of a different hypothesis. Data can be assigned to different populations only after the statistical tests have been made. Trimming the extremes from the data reduces the variance (Table 1) but this makes it impossible to apply the *F*-test.

The CLT and the Law of Large Numbers state [4–7] that various size samples from a heterogeneous population will yield the same mean but will have variances that decrease as n or V, as the number n of samples increases or as the volume, V, of the sampled region increases. If OIB are small volume samples and MORB are large volume samples from the same reservoir then the variance of OIB will be greater than the variance of MORB. Small samples are more likely to have extreme values than samples that blend components from a large volume.

If two datasets have the same mean and different variances they may be different size samples from the same distribution. A consequence of the CLT is that the ratio of the variances is inversely proportional to the ratios of the sampled volumes. The *F*-parameter can then be interpreted as the ratios of sample volumes [7]. The inferred volume sampled by MORB1 is 5% greater than the volume sampled by oceanic islands (Table 1). The larger volume sampled by ridges includes the effect of larger degrees of partial melting as well as larger physical volumes associated with ridge magmatism. Variances also reflect the efficiency of melt–crystal partitioning, and of degassing and gas retention.

Table 1
Statistics of $^3He/^4He$ ratios (R) normalized to the ratio in air (R_A) for MORB and OIB samples (n samples in the population)

	Mean ± S.D.	n	Note[a]
MORB1	9.14 ± 3.59	503	1
MORB2	8.58 ± 1.81	576	2
MORB3	8.11 ± 1.41	238	3
MORB4	8.22 ± 0.85	206	4
OIB	7.67 ± 3.68	23	5

[a] 1. [4]. Unfiltered data from the global spreading ridge system. 2. ([4]; D. Graham, personal communication (2000), in [4]). Submarine samples from the crest of the global midocean ridge system, potentially contaminated samples removed. 3. [4,5]. Strongly filtered data; depths <2.5 km, $R/R_A>11$, removed. 4. [2]. MORB with 'plume influenced' samples removed. 5. [2].

4. Scale of heterogeneity

Chemical heterogeneity probably exists at all scales, from the grain scale to the hemisphere scale. Some oceanic islands have data variances that are smaller than the global spreading ridge data, suggesting that an important scale length in the mantle is tens to hundreds of kilometers. Individual volcanoes at any point in time may not have access to the total population. These scales are large enough to avoid diffusive homogenization. When taken all together the global hotspot dataset defines a distribution that is statistically the same as the global spreading ridge dataset. Helium data for individual islands and groups of islands support the conclusion reached here [4,5]. There is some indication of a temporal evolution in isotopic heterogeneity, suggesting a depth dependence [4,5].

5. Averaging

In geochemical studies, ratios are often treated the same as absolute quantities in forming the statistics and comparing hotspot and ridge samples. The stable statistics for ratios are the median and the geometric mean. When helium ratios are averaged they should be weighted by the helium abundance {He}. If the mantle contains a range of R values the region with the highest {He} will dominate the isotopic ratio of the blend [5]. The lowest {He} samples can only be detected by small-scale sampling, not by melting of large volumes. The highest and lowest R tend to be found in small-scale dense sampling programs and these samples tend to have low {He}.

6. Discussion

Allègre et al. [2] have shown that the variance of MORB values for R depends on spreading rate. The slowest spreading ridge systems have the largest variance, consistent with the CLT. This trend continues for ultra-slow ridges such as Iceland, Red Sea, Ethiopia and backarc basins [4]. The standard deviation of the OIB global dataset is about four times that of the slowest spreading ridge tabulated in Allègre et al. (SW Indian Ridge) and comparable to the ultra-slow ridges tabulated elsewhere [4,5]. The implication is that the SW Indian Ridge processes about 16 times more mantle (larger volume and/or more melting) than the average oceanic island, and about four times more mantle than the global submarine dataset (MORB2). The alternative is that some ridge segments are more homogeneous than others because of prior history of subduction and melt extraction. Statistical methods could be used to test this hypothesis. The high variance and extreme values, including high R samples, found along some slow spreading ridges and subaerial rifts may simply be a continuation of the trend found along mature ridges, and reflect sampling statistics rather than independent sources.

Basalts and hydrothermal fluids having ^{3}He/^{4}He ratios greater than about 8.5–9.0 times the atmospheric value are commonly believed to be 'plume-type' or 'lower-mantle' helium since these ratios are *significantly higher than the normal MORB range* [9–11]. This idea, however, is not supported by statistical tests applied in this paper and elsewhere [4,5].

7. Chemical stratification

The standard model of mantle noble gas geochemistry [1,3] divides the mantle into a depleted degassed upper mantle homogenized by convection, and an undegassed primordial lower mantle that is not well stirred. This model is based on a string of assumptions:

1. ^{3}He/^{4}He ratios higher than the MORB average reflect high ^{3}He abundances;
2. High *inferred* (not measured) ^{3}He abundances imply a primordial undegassed reservoir, or a reservoir more primitive and less outgassed than the upper mantle;
3. This primordial reservoir must be isolated from the upper mantle;
4. It is therefore deep and is the lower mantle;
5. This deep isolated reservoir can be tapped by plumes, which deliver material to the surface at oceanic islands.

The various paradoxes associated with these models, and the alternatives to each of the assumptions, were discussed in previous papers [4,5,12]. One of these paradoxes is the very low ^{3}He content of most OIB. This is generally attributed to pre-eruptive degassing but this is inconsistent with He/Ne ratios and with global He–balance calculations [4,12].

Complex and hybrid geochemical [3,10,11] and geophysical models [13,14] have been devised to rationalize the assumed coexistence of *a depleted degassed well-stirred* upper mantle and a heterogeneous *undegassed primordial* lower mantle, assuming that OIB and MORB represent distinct isolated reservoirs, and that the upper mantle can only provide homogeneous MORB-like materials. Mass–balance calculations show that the MORB reservoir need occupy only a fraction of the upper mantle [15]. The OIB components probably also reside at shallow depths [4,12,15]. The buoyant and refractory parts of the shallow mantle and neutrally buoyant or small blobs can also be isolated. The present paper shows that the basic premise of the two reservoir model is false. There is no statistical difference between basalts from the global spreading ridge system and hotspot basalts. It is statistically invalid to compare maximum values in one population with means of another population, in which the extreme values have been trimmed.

The present results favor a sampling explanation, rather than an isolated reservoir explanation, for the statistics of helium isotopes. Components which are distinct in He/(U+Th) ratios, age or history are, of course, still required but the mantle may be more like a marble cake or a plum pudding, or have small-scale stratification, than one characterized by large isolated reservoirs separated by seismic discontinuities.

Recent calculations make the idea of chemical stratification much more plausible than previously thought [16–19]. Whole mantle convection simulations have been unable to account for plate tectonics, deep mantle tomography, dynamic topography and the spectral and spatial patterns of tomographic models [18,19,21]. In contrast, thermochemical and stratified mantle simulations account for both the geoid and dynamic topography, and explain deep mantle features as well as the change in spectral properties with depth [19–21]. Layered convection also accounts for mid-mantle features [20] which have been widely accepted as evidence for whole mantle convection [21]. Large-scale major element chemical stratification of the mantle need have nothing to do with chemical heterogeneity of mantle samples. It is only by assumption, and then convention, that the upper and lower mantles have been treated as the sources of various components in basalts in box models of mantle evolution. The seismic discontinuities which define the subdivisions of the mantle are mainly phase changes, not chemical boundaries which are much more subtle. Geochemistry and statistics are mute on the location, size and configuration of the inhomogeneities sampled by volcanoes. Although the deep mantle may be isolated from the shallow mantle by density or viscosity [15,17,19,20] it may also be isolated from surface volcanoes.

The outermost 300 km of the mantle is extremely heterogeneous. This is the low-viscosity region where slabs dehydrate and the melts for surface volcanoes originate [15]. Slabs, presumably stripped of sediments and hydrous minerals and most of the incompatible elements, descend to 650 km where they stagnate [18]. Ancient slabs may have penetrated to 1000 km depth, the base of the upper mantle sensu stricto. Although the debate about whole mantle convection and deep slab penetration (below 1000 km) continues [13,14,18–23], it is only by convention that the deep mantle is treated as an important and accessible geochemical reservoir [3]. Mass balance does not require this [15].

It is easier to isolate components and evolve isotopic anomalies in the shallower colder parts of Earth – crust, sediments, lithosphere, tectosphere, perisphere, slabs – than in the deeper hotter regions but it is often assumed that these components must be recycled to the lower mantle before they become available to surface volcanoes. Some slabs may sink to 1000 km depth but the geochemically useful tracers appear to reside in phases that leave the slab at shallow depths. Likewise, the gases that exsolve from ascending magmas are isolated at shallow depths and can be stored for some time in refractory phases or layers with different He/(U,Th) ratios than the source, the degassed magma or the residual magma.

8. Conclusion

The statistics (means, medians) of many hotspot datasets are identical or similar to midocean ridge statistics ([2,4,6]; D. Graham, personal communication (2000), in [4]). The hypothesis that OIB and MORB are drawn from the same population cannot be rejected. The differences in variances of the datasets are consistent with midocean ridges sampling larger volumes than most oceanic islands. This is an extension of the idea that fast spreading ridges sample larger volumes of the mantle than slow spreading ridges and therefore have smaller variances of various geochemical parameters [2].

The ongoing debate about whole-mantle vs. layered-mantle convection [15,24] and deep slab penetration may have nothing to do with the chemical inhomogeneity of the mantle as sampled by basalts and xenoliths.

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